

Giant magnon and spike solutions with two spins in $AdS_4 \times CP^3$

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ABSTRACT: In the string theory in $AdS_4 \times CP^3$ we construct the giant magnon and spike solutions with two spins in two kinds of subspaces of $R_t \times CP^3$ and derive the dispersion relations for them. For the single giant magnon solution in one subspace we show that its dispersion relation is associated with that of the big one-spin giant magnon solution in the RP^2 subspace. For the single giant magnon solution in the other complementary subspace its dispersion relation is similar to that of the one-spin giant magnon solution living in the S^2 subspace but has one additional spin dependence.

KEYWORDS: AdS-CFT Correspondence, Bosonic Strings.

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1. Introduction

There has been an exciting subject toward understanding the correspondence between the four-dimensional $\mathcal{N} = 4$ super Yang-Mills theory and the type IIB superstring in $AdS_5 \times S^5$ [1]. Recently, inspired by the study of Bagger, Lambert and Gustavsson about the three-dimensional $\mathcal{N} = 8$ supersymmetric theory for the multiple M2-branes [2], Aharony, Bergman, Jafferis and Maldacena (ABJM) have constructed a $\mathcal{N} = 6$ superconformal $SU(N) \times SU(N)$ Chern-Simons theory in three dimensions at level $(k, -k)$ coupled with bi-fundamental matter as a world-volume theory of N coincident M2-branes on orbifold C^4/Z_k [3]. When the 't Hooft coupling constant defined by $\lambda = N/k$ becomes large and $1 \ll N \ll k^5$, the M-theory with N -units of four-form flux on $AdS_4 \times S^7/Z_k$ can be compactified to the type IIA superstring theory in $AdS_4 \times CP^3$.

The new correspondence between the ABJM theory and the type IIA superstring theory in $AdS_4 \times CP^3$ has been studied by analyzing the string spectrum in the Penrose limit of this IIA background [4–6] and by finding an integrable Hamiltonian of an $SU(4)$ spin chain [7, 6] with sites alternating between the fundamental and anti-fundamental representations and further constructing two-loop Bethe equation [7, 8]. The all-loop Bethe ansatz equation has been obtained [9] by developing the perturbative result [7] and the classical integrability in the strong coupling which was shown by deriving a supercoset sigma model for the type IIA superstring theory in $AdS_4 \times CP^3$ [10] and an $OSp(2, 2|6)$ symmetric finite gap algebraic curve [11]. This all-loop Bethe ansatz equation has been also produced by analyzing the S-matrix [12].

There have been a construction of a giant magnon solution with one angular momentum in the $SU(2) \times SU(2)$ sector of type IIA string theory in $AdS_4 \times CP^3$ [5] (see also [6]) and a computation of its finite-size correction [13], where the string moving in $R_t \times S^2 \times S^2$ has opposite azimuthal angles in the two S^2 . The dispersion relation of the spike solution with one spin in $R_t \times S^2 \times S^2$ and its finite-size correction have been derived [14]. The circular, folded and pulsating string solutions in the $SU(2) \times SU(2)$ sector have been studied [15].

Further, the giant magnon and spike solutions with two angular momenta have been constructed by reducing the string dynamics on $AdS_4 \times CP^3$ to the Neumann-Rosochatius integrable system [16], where the dispersion relations and the finite-size corrections for them have been computed. On the other hand from the analysis of the finite gap algebraic curve the dispersion relation of the giant magnon and the one-loop quantum correction have been presented [17]. The $AdS_4 \times CFT_3$ duality has been further investigated from various view points [18–25].

In ref. [16] the four embedding complex coordinates constrained by a CP^3 condition have been used for parametrizing the CP^3 space to construct the string moving in $R_t \times S^3 \times S^3$ with two angular momenta in the first S^3 and with exactly opposite angular momenta in the second S^3 , and extract the dispersion relation of a single giant magnon solution living in S^3 with two angular momenta. We will choose the spherical coordinates of CP^3 space presented in ref. [26, 4] to construct a single giant magnon solution as well as a single spike solution with two angular momenta living in one subspace of CP^3 and derive the dispersion relations for them where the dispersion relation for the giant magnon becomes the same expression as that for the two giant magnons living in the two S^3 subspaces in ref. [16]. By considering the other complementary subspace of CP^3 we will obtain some different types of giant magnon and spike solutions with two angular momenta.

2. Giant magnon and spike solutions in one subspace of CP^3

Following the prescriptions to construct the giant magnon solutions [27–29] and the spike solutions [30, 31] in $AdS_5 \times S^5$, we consider the string states such as a giant magnon and a spike in $AdS_4 \times CP^3$ which have two angular momenta in CP^3 . The explicit form of metric on $AdS_4 \times CP^3$ is written by [26, 4]

$$\begin{aligned}
 ds^2 = & \frac{R^2}{4} [-\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\Omega_2^2] \\
 & + R^2 \left[d\xi^2 + \cos^2 \xi \sin^2 \xi \left(d\psi + \frac{1}{2} \cos \theta_1 d\phi_1 - \frac{1}{2} \cos \theta_2 d\phi_2 \right)^2 \right. \\
 & \left. + \frac{1}{4} \cos^2 \xi (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + \frac{1}{4} \sin^2 \xi (d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2) \right], \quad (2.1)
 \end{aligned}$$

where $0 \leq \xi \leq \pi/2$, $-2\pi \leq \psi \leq 2\pi$ and (θ_i, ϕ_i) are coordinates of two S^2 's. The radius R is given by $R^2 = 2^{5/2} \pi \lambda^{1/2}$ with the 't Hooft coupling constant $\lambda = N/k$. We are interested in a string configuration in $R_t \times CP^3$ which is parametrized by $\rho = 0$ as well as $\theta_1 = \theta_2 \equiv \theta$.

Here with this parametrization we write down the Polyakov action in the conformal gauge

$$\begin{aligned}
 S = & \sqrt{2\lambda} \int d\tau d\sigma \left[-\frac{1}{4} \dot{t}^2 + \dot{\xi}^2 - \dot{\xi}'^2 + \cos^2 \xi \sin^2 \xi (\dot{\psi}^2 - \dot{\psi}'^2) \right. \\
 & \left. + \frac{1}{4} (\dot{\theta}^2 - \dot{\theta}'^2) + \cos^2 \xi \sin^2 \xi \cos \theta \left(\dot{\psi}(\dot{\phi}_1 - \dot{\phi}_2) - \dot{\psi}'(\phi'_1 - \phi'_2) \right) \right]
 \end{aligned}$$

$$\begin{aligned}
& +\frac{1}{4}\cos^2\xi(\sin^2\theta+\cos^2\theta\sin^2\xi)(\dot{\phi}_1^2-\phi_1'^2)+\frac{1}{4}\sin^2\xi(\sin^2\theta+\cos^2\theta\cos^2\xi)(\dot{\phi}_2^2-\phi_2'^2) \\
& -\frac{1}{2}\cos^2\xi\sin^2\xi\cos^2\theta(\dot{\phi}_1\dot{\phi}_2-\phi_1'\phi_2')\Big]. \tag{2.2}
\end{aligned}$$

The equation of motion for θ is given by

$$\begin{aligned}
\ddot{\theta}-\theta''-\sin\theta\cos\theta[(\cos^2\xi\dot{\phi}_1+\sin^2\xi\dot{\phi}_2)^2-(\cos^2\xi\phi_1'+\sin^2\xi\phi_2')^2] \\
+2\cos^2\xi\sin^2\xi\sin\theta[\dot{\psi}(\dot{\phi}_1-\dot{\phi}_2)-\psi'(\phi_1'-\phi_2')] = 0, \tag{2.3}
\end{aligned}$$

while the equation of motion for ψ reads

$$\begin{aligned}
2[\partial_\tau(\cos^2\xi\sin^2\xi\dot{\psi})-\partial_\sigma(\cos^2\xi\sin^2\xi\psi')] + \partial_\tau[\cos^2\xi\sin^2\xi\cos\theta(\dot{\phi}_1-\dot{\phi}_2)] \\
-\partial_\sigma[\cos^2\xi\sin^2\xi\cos\theta(\phi_1'-\phi_2')] = 0. \tag{2.4}
\end{aligned}$$

From the two equations we note that there is an obvious solution

$$\theta = \frac{\pi}{2}, \quad \psi = 0. \tag{2.5}$$

Then the relevant string action is

$$S = \sqrt{2\lambda} \int d\tau d\sigma \left[-\frac{1}{4}\dot{t}^2 + \xi^2 - \xi'^2 + \frac{1}{4}\cos^2\xi(\dot{\phi}_1^2 - \phi_1'^2) + \frac{1}{4}\sin^2\xi(\dot{\phi}_2^2 - \phi_2'^2) \right], \tag{2.6}$$

where τ and σ range from $-\infty$ to ∞ .

Let us make the following ansatz for a rotating string soliton with two spins

$$t = \kappa\tau, \quad \xi = \xi(\eta), \quad \phi_1 = \omega_1\tau + f_1(\eta), \quad \phi_2 = \omega_2\tau + f_2(\eta), \tag{2.7}$$

where $\eta = \alpha\sigma + \beta\tau$. The equations of motion for ϕ_1, ϕ_2 lead to

$$\partial_\eta f_1 = \frac{1}{\alpha^2 - \beta^2} \left(\frac{C_1}{\cos^2\xi} + \beta\omega_1 \right), \quad \partial_\eta f_2 = \frac{1}{\alpha^2 - \beta^2} \left(\frac{C_2}{\sin^2\xi} + \beta\omega_2 \right), \tag{2.8}$$

where C_1 and C_2 are integration constants. The Virasoro constraints $T_{\tau\tau} + T_{\sigma\sigma} = 0, T_{\tau\sigma} = 0$ yield

$$\begin{aligned}
-\frac{\kappa^2}{4(\alpha^2 + \beta^2)} + \xi'^2 + \frac{\cos^2\xi}{4} \left(f_1'^2 + \frac{\omega_1^2 + 2\omega_1\beta f_1'}{\alpha^2 + \beta^2} \right) + \frac{\sin^2\xi}{4} \left(f_2'^2 + \frac{\omega_2^2 + 2\omega_2\beta f_2'}{\alpha^2 + \beta^2} \right) = 0, \\
\xi'^2 + \frac{\cos^2\xi}{4} \left(f_1'^2 + \frac{\omega_1 f_1'}{\beta} \right) + \frac{\sin^2\xi}{4} \left(f_2'^2 + \frac{\omega_2 f_2'}{\beta} \right) = 0, \tag{2.9}
\end{aligned}$$

where the prime here denotes the derivative with respect to η . The difference of these two constraints gives a relation between few parameters

$$\beta\kappa^2 + \omega_1 C_1 + \omega_2 C_2 = 0. \tag{2.10}$$

We restrict ourselves to $C_1 = 0$ for constructing a giant magnon solution and a spike solution so that C_2 is given by

$$C_2 = -\frac{\beta\kappa^2}{\omega_2}. \tag{2.11}$$

Combining together we have

$$\xi'^2 = \frac{1}{4(\alpha^2 - \beta^2)^2} \left[\kappa^2(\alpha^2 + \beta^2) - \frac{C_2^2}{\sin^2 \xi} - \alpha^2(\omega_1^2 \cos^2 \xi + \omega_2^2 \sin^2 \xi) \right], \quad (2.12)$$

which is the first integral of the equation of motion for ξ . For $\omega_2 > \omega_1$ this equation is expressed in the following form

$$\xi' = \pm \frac{\alpha \sqrt{\omega_2^2 - \omega_1^2}}{2(\alpha^2 - \beta^2) \sin \xi} \sqrt{(\cos^2 \xi_+ - \cos^2 \xi)(\cos^2 \xi - \cos^2 \xi_-)}, \quad (2.13)$$

where the + and - signs correspond to $\alpha^2 > \beta^2$ and $\alpha^2 < \beta^2$ respectively and

$$\begin{aligned} \cos^2 \xi_{\pm} = & \frac{1}{2} \left[\frac{\alpha^2(2\omega_2^2 - \omega_1^2) - \kappa^2(\alpha^2 + \beta^2)}{\alpha^2(\omega_2^2 - \omega_1^2)} \right. \\ & \left. \pm \sqrt{\left(\frac{\alpha^2(2\omega_2^2 - \omega_1^2) - \kappa^2(\alpha^2 + \beta^2)}{\alpha^2(\omega_2^2 - \omega_1^2)} \right)^2 - 4 \frac{\alpha^2\omega_2^2 + C_2^2 - \kappa^2(\alpha^2 + \beta^2)}{\alpha^2(\omega_2^2 - \omega_1^2)}} \right] \end{aligned} \quad (2.14)$$

with $\cos \xi_+ > \cos \xi_-$.

The string energy is provided by

$$E = \pm 2\sqrt{2\lambda} \frac{\kappa(\alpha^2 - \beta^2)}{\alpha^2 \sqrt{\omega_2^2 - \omega_1^2}} \int_{\xi_+}^{\xi_-} d\xi \frac{\sin \xi}{\sqrt{(\cos^2 \xi_+ - \cos^2 \xi)(\cos^2 \xi - \cos^2 \xi_-)}}, \quad (2.15)$$

while the spins J_1 and J_2 associated with the angular variables ϕ_1 and ϕ_2 are expressed as

$$\begin{aligned} J_1 = & \pm 2\sqrt{2\lambda} \frac{\alpha^2 - \beta^2}{\alpha^2 \sqrt{\omega_2^2 - \omega_1^2}} \int_{\xi_+}^{\xi_-} d\xi \frac{\sin \xi \cos^2 \xi}{\sqrt{(\cos^2 \xi_+ - \cos^2 \xi)(\cos^2 \xi - \cos^2 \xi_-)}} \left(\omega_1 + \frac{\beta^2 \omega_1}{\alpha^2 - \beta^2} \right), \\ J_2 = & \pm 2\sqrt{2\lambda} \frac{\alpha^2 - \beta^2}{\alpha^2 \sqrt{\omega_2^2 - \omega_1^2}} \int_{\xi_+}^{\xi_-} d\xi \frac{\sin^3 \xi}{\sqrt{(\cos^2 \xi_+ - \cos^2 \xi)(\cos^2 \xi - \cos^2 \xi_-)}} \\ & \times \left[\omega_2 + \frac{\beta}{\alpha^2 - \beta^2} \left(\frac{C_2}{\sin^2 \xi} + \beta \omega_2 \right) \right]. \end{aligned} \quad (2.16)$$

The angle difference for ϕ_2 is also described by

$$\begin{aligned} \Delta\phi_2 = & - \int d\phi_2 = - \int d\xi \frac{\phi_2'}{\xi'} \\ = & \mp \frac{4}{\alpha \sqrt{\omega_2^2 - \omega_1^2}} \int_{\xi_+}^{\xi_-} d\xi \frac{\sin \xi}{\sqrt{(\cos^2 \xi_+ - \cos^2 \xi)(\cos^2 \xi - \cos^2 \xi_-)}} \left(\frac{C_2}{\sin^2 \xi} + \beta \omega_2 \right), \end{aligned} \quad (2.17)$$

where we use a minus sign to make the angle difference positive. The giant magnon and spike solutions in the infinite volume can be constructed by taking a limit $\xi_- \rightarrow \pi/2$. Therefore the infinite volume limit specified by $\cos^2 \xi_- = 0$ in (2.14) yields a relation

$$\kappa^2(\alpha^2 + \beta^2) = \alpha^2\omega_2^2 + C_2^2, \quad (2.18)$$

which becomes the following equation through (2.11)

$$(\kappa^2 - \omega_2^2) \left(\alpha^2 - \frac{\kappa^2 \beta^2}{\omega_2^2} \right) = 0. \quad (2.19)$$

There are two solutions $\kappa = \omega_2$ and $\kappa = \alpha\omega_2/\beta$, which are associated with the giant magnon and spike solutions respectively. From (2.14), $\sin^2 \xi_+$ for each solution is evaluated as

$$\sin^2 \xi_+ = \frac{\omega_2^2 \beta^2}{(\omega_2^2 - \omega_1^2) \alpha^2} \quad (\text{giant magnon}), \quad \sin^2 \xi_+ = \frac{\omega_2^2 \alpha^2}{(\omega_2^2 - \omega_1^2) \beta^2} \quad (\text{spike}) \quad (2.20)$$

so that $\alpha^2 > \beta^2$ and $\alpha^2 < \beta^2$ correspond to the giant magnon and spike solutions respectively.

For the giant magnon solution case the difference between the infinite energy E and the infinite spin J_2 becomes finite as

$$E - J_2 = 2\sqrt{2\lambda} \frac{\omega_2 \cos \xi_+}{\sqrt{\omega_2^2 - \omega_1^2}}, \quad (2.21)$$

which is also expressed as $E - J_2 = J_1 \omega_2 / \omega_1$ in terms of the finite spin J_1 . The elimination of ω_2 / ω_1 leads to

$$E - J_2 = \sqrt{J_1^2 + 8\lambda \cos^2 \xi_+}. \quad (2.22)$$

Since $C_2 / \sin^2 \xi + \beta \omega_2$ in (2.17) becomes $-\beta \omega_2 \cos^2 \xi / \sin^2 \xi$, the angle difference is given by a positive finite value

$$\begin{aligned} \Delta\phi_2 &= 4 \sin \xi_+ \int_{\xi_+}^{\pi/2} d\xi \frac{\cos \xi}{\sin \xi \sqrt{\sin^2 \xi - \sin^2 \xi_+}} \\ &= 4 \left(\frac{\pi}{2} - \xi_+ \right). \end{aligned} \quad (2.23)$$

The magnon momentum p is identified with the angle difference $\Delta\phi_2$ so that we get a dispersion relation of a single giant magnon in one subspace of $R_t \times CP^3$

$$E - J_2 = \sqrt{J_1^2 + 8\lambda \sin^2 \frac{p}{4}}, \quad (2.24)$$

which is of a similar form to that of the dyonic giant magnon in $AdS_5 \times S^5$ [32]. In ref. [16] by analyzing the string motion in $R_t \times S^3 \times S^3$ as the Neumann-Rosochatius integrable system the dispersion relation of two-spin giant magnon was presented as

$$E - J_2 = \sqrt{J_1^2 + 8\lambda \sin^2 \frac{\tilde{p}}{2}}. \quad (2.25)$$

This string configuration expressed by the four embedding complex coordinates is the same as our string solution specified by the spherical coordinates. Because of the slight different definition of the angle variable corresponding to ϕ_2 by a factor 1/2 the magnon momentum \tilde{p} is identified with $p/2$, so that (2.24) agrees with (2.25). By regarding (2.25) as the dispersion relation of two two-spin giant magnons living in the two S^3 subspaces, the following equation was extracted as the dispersion relation of a single two-spin giant magnon living in the S^3 subspace

$$E - J_2 = \sqrt{\frac{J_1^2}{4} + 2\lambda \sin^2 \frac{\tilde{p}}{2}}. \quad (2.26)$$

When J_1 is turned off the dispersion relation (2.26) reduces to that of a single one-spin giant magnon living in the S^2 subspace [5]. When the result (2.24) with a dependence of $\sin^2 p/4$ is regarded for a single magnon solution, it shows the same expression as the dispersion relation for a single big giant magnon solution with two spins in $AdS_4 \times CP^3$ in ref. [17] (see [6]), where it was derived by using the algebraic curve technique [33, 34], and the other dispersion relation (2.26) was also computed separately.

Let us consider the single spike solution case. Since $C_2/\sin^2 \xi + \beta\omega_2$ in (2.17) is expressed by $\omega_2(\beta^2 \sin^2 \xi - \alpha^2)/\beta \sin^2 \xi$ which is positive for $\pi/2 > \xi > \xi_+$ owing to $\beta^2 \sin^2 \xi_+ - \alpha^2 = \alpha^2 \omega_1^2 / (\omega_2^2 - \omega_1^2) > 0$. The angle difference $\Delta\phi_2$ becomes positive divergent. The difference between the infinite energy E and the infinite positive angle difference $\Delta\phi_2$ multiplied with $\sqrt{2\lambda}/2$ turns out to be finite

$$\begin{aligned} E - \frac{1}{2}\sqrt{2\lambda}\Delta\phi_2 &= 2\sqrt{2\lambda} \sin \xi_+ \int_{\xi_+}^{\pi/2} d\xi \frac{\cos \xi}{\sin \xi \sqrt{\sin^2 \xi - \sin^2 \xi_+}} \\ &= 2\sqrt{2\lambda} \bar{\xi} \end{aligned} \tag{2.27}$$

with $\bar{\xi} = \pi/2 - \xi_+$. On the other hand the two spins J_1 and J_2 are finite as expressed by

$$\begin{aligned} J_1 &= -2\sqrt{2\lambda} \frac{\omega_1 \cos \xi_+}{\sqrt{\omega_2^2 - \omega_1^2}}, \\ J_2 &= 2\sqrt{2\lambda} \frac{\omega_2 \cos \xi_+}{\sqrt{\omega_2^2 - \omega_1^2}}, \end{aligned} \tag{2.28}$$

which satisfy a relation

$$J_2 = \sqrt{J_1^2 + 8\lambda \sin^2 \bar{\xi}}. \tag{2.29}$$

3. Giant magnon and spike solutions in the complementary subspace of CP^3

We construct the other string solution by making the following ansatz

$$\begin{aligned} t &= \kappa\tau, & \theta &= \frac{\pi}{2}, & \psi &= \omega\tau + f(\eta), \\ \xi &= \xi(\eta), & \phi_1 &= \phi_2 \equiv \phi \end{aligned} \tag{3.1}$$

with $\phi = \nu\tau$. The equation of motion for θ (2.3) is solved by $\theta = \pi/2$ and $\phi_1 = \phi_2$, which is a complementary choice to (2.5). The equation of motion for ϕ is simply satisfied, while the equation of motion for ψ (2.4) yields

$$f' = \frac{1}{\alpha^2 - \beta^2} \left(\frac{C}{\sin^2 2\xi} + \beta\omega \right) \tag{3.2}$$

with an integration constant C . The Virasoro constraints read

$$\begin{aligned} -\frac{\kappa^2}{4(\alpha^2 + \beta^2)} + \xi'^2 + \frac{\sin^2 2\xi}{4} \left(f'^2 + \frac{\omega^2 + 2\omega\beta f'}{\alpha^2 + \beta^2} \right) + \frac{\nu^2}{4(\alpha^2 + \beta^2)} &= 0, \\ \xi'^2 + \frac{\sin^2 2\xi}{4} \left(f'^2 + \frac{\omega f'}{\beta} \right) &= 0, \end{aligned} \tag{3.3}$$

whose difference provides a relation $\beta(\kappa^2 - \nu^2) + \omega C = 0$. We gather them together to have

$$\xi'^2 = \frac{1}{4(\alpha^2 - \beta^2)^2} \left[(\kappa^2 - \nu^2)(\alpha^2 + \beta^2) - \frac{C^2}{\sin^2 2\xi} - \alpha^2 \omega^2 \sin^2 2\xi \right], \quad (3.4)$$

which yields

$$\xi' = \pm \frac{\alpha\omega}{2(\alpha^2 - \beta^2) \sin 2\xi} \sqrt{(\cos^2 2\xi_+ - \cos^2 2\xi)(\cos^2 2\xi - \cos^2 2\xi_-)}, \quad (3.5)$$

where the + and - signs correspond to the giant magnon ($\alpha^2 > \beta^2$) and the spike ($\alpha^2 < \beta^2$) solutions and

$$\begin{aligned} \cos^2 2\xi_{\pm} = & \frac{1}{2} \left[\frac{2\alpha^2\omega^2 - (\kappa^2 - \nu^2)(\alpha^2 + \beta^2)}{\alpha^2\omega^2} \right. \\ & \left. \pm \sqrt{\left(\frac{2\alpha^2\omega^2 - (\kappa^2 - \nu^2)(\alpha^2 + \beta^2)}{\alpha^2\omega^2} \right)^2 - 4 \frac{\alpha^2\omega^2 + C^2 - (\kappa^2 - \nu^2)(\alpha^2 + \beta^2)}{\alpha^2\omega^2}} \right]. \end{aligned} \quad (3.6)$$

The rotating string is specified by the following energy and two spins J_ϕ and J_ψ associated with the ϕ and ψ directions

$$\begin{aligned} E &= \pm 2\sqrt{2\lambda} \frac{\kappa(\alpha^2 - \beta^2)}{\alpha^2\omega} \int_{\xi_+}^{\xi_-} d\xi \frac{\sin 2\xi}{\sqrt{(\cos^2 2\xi_+ - \cos^2 2\xi)(\cos^2 2\xi - \cos^2 2\xi_-)}}, \\ J_\phi &= \pm 2\sqrt{2\lambda} \frac{\nu(\alpha^2 - \beta^2)}{\alpha^2\omega} \int_{\xi_+}^{\xi_-} d\xi \frac{\sin 2\xi}{\sqrt{(\cos^2 2\xi_+ - \cos^2 2\xi)(\cos^2 2\xi - \cos^2 2\xi_-)}}, \\ J_\psi &= \pm 2\sqrt{2\lambda} \frac{\alpha^2 - \beta^2}{\alpha^2\omega} \int_{\xi_+}^{\xi_-} d\xi \frac{\sin^3 2\xi}{\sqrt{(\cos^2 2\xi_+ - \cos^2 2\xi)(\cos^2 2\xi - \cos^2 2\xi_-)}} \\ &\quad \times \left[\omega + \frac{\beta}{\alpha^2 - \beta^2} \left(\frac{C}{\sin^2 2\xi} + \beta\omega \right) \right]. \end{aligned} \quad (3.7)$$

The angle difference for ψ is given by

$$\begin{aligned} \Delta\psi &= - \int d\psi \\ &= \mp \frac{4}{\alpha\omega} \int_{\xi_+}^{\xi_-} d\xi \frac{\sin 2\xi}{\sqrt{(\cos^2 2\xi_+ - \cos^2 2\xi)(\cos^2 2\xi - \cos^2 2\xi_-)}} \left(\frac{C}{\sin^2 2\xi} + \beta\omega \right). \end{aligned} \quad (3.8)$$

Here we take an infinite volume limit $\xi_- \rightarrow \pi/4$ in (3.6) to obtain an equation

$$(\kappa^2 - \nu^2 - \omega^2) \left(\alpha^2 - \frac{\beta^2(\kappa^2 - \nu^2)}{\omega^2} \right) = 0, \quad (3.9)$$

whose solutions $\kappa^2 - \nu^2 = \omega^2$ and $\kappa^2 - \nu^2 = \alpha^2\omega^2/\beta^2$ specify the giant magnon and spike solutions in the infinite volume respectively. From (3.6) each solution is characterized by

$$\sin^2 2\xi_+ = \frac{\beta^2}{\alpha^2} \text{ (giant magnon),} \quad \sin^2 2\xi_+ = \frac{\alpha^2}{\beta^2} \text{ (spike),} \quad (3.10)$$

which implies that $\alpha^2 > \beta^2$ and $\alpha^2 < \beta^2$ are associated with the giant magnon and spike solutions respectively.

We begin to consider the giant magnon solution case. In view of the expressions in (3.7) we find one relation between the energy E and the spins J_ϕ and J_ψ in the infinite volume

$$\begin{aligned} \kappa E - \nu J_\phi - \omega J_\psi &= 2\sqrt{2\lambda}\omega \int_{\xi_+}^{\pi/4} d\xi \frac{\sin 2\xi \cos 2\xi}{\sqrt{\cos^2 2\xi_+ - \cos^2 2\xi}} \\ &= \sqrt{2\lambda}\omega \cos 2\xi_+, \end{aligned} \tag{3.11}$$

where the logarithmic divergences of E, J_ϕ and J_ψ are canceled out under the suitably chosen coefficients. There is the other relation

$$\frac{E}{\kappa} = \frac{J_\phi}{\nu}. \tag{3.12}$$

From (3.8) the angle difference identified with the magnon momentum p is given by

$$p = \Delta\psi = 2 \left(\frac{\pi}{2} - 2\xi_+ \right). \tag{3.13}$$

Combining (3.11), (3.12) and (3.13) with

$$\left(\frac{\kappa}{\omega} \right)^2 - \left(\frac{\nu}{\omega} \right)^2 = 1 \tag{3.14}$$

to eliminate the parameters we get the following dispersion relation for this rotating two-spin string solution

$$\sqrt{E^2 - J_\phi^2} - J_\psi = \sqrt{2\lambda} \sin \frac{p}{2}. \tag{3.15}$$

In the $J_\phi = 0$ case this giant magnon-like dispersion relation reduces to the same expression as that for a single one-spin giant magnon living in the S^2 subspace extracted from the giant magnon solution in the $SU(2) \times SU(2)$ sector which consists of two giant magnons, one for each $SU(2)$ with opposite momenta [5]. There were investigations of the giant magnon string solutions in various backgrounds [35, 36]. The giant magnon-like expression in (3.15) takes a similar form to the dispersion relation in ref. [35], where the giant magnon solution in the near horizon geometry of the NS 5-brane background was constructed to be expressed in terms of several charges.

For the spike solution case we produce the following relation between the logarithmic divergent quantities such as the energy E , the spin J_ϕ and the angle difference $\Delta\psi$

$$\begin{aligned} \kappa E - \nu J_\phi - \frac{\omega\alpha}{\beta} \frac{\sqrt{2\lambda}}{2} \Delta\psi &= 2\sqrt{2\lambda} \frac{\omega\alpha^2}{\beta^2} \int_{\xi_+}^{\pi/4} d\xi \frac{\cos 2\xi}{\sqrt{\cos^2 2\xi_+ - \cos^2 2\xi} \sin 2\xi} \\ &= \sqrt{2\lambda} \frac{\omega\alpha}{\beta} \tilde{\xi}, \end{aligned} \tag{3.16}$$

where $\tilde{\xi} = \pi/2 - 2\xi_+$ and the logarithmic divergences are suitably canceled out. We use two relations (3.12) and $\kappa^2 - \nu^2 = \omega^2\alpha^2/\beta^2$ to eliminate the parameters and rewrite (3.16) as

$$\sqrt{E^2 - J_\phi^2} - \frac{1}{2}\sqrt{2\lambda}\Delta\psi = \sqrt{2\lambda}\tilde{\xi}, \tag{3.17}$$

while the other spin J_ψ is evaluated as

$$J_\psi = \sqrt{2\lambda} \sin \tilde{\xi}. \quad (3.18)$$

The spike-like dispersion relation (3.17) has a common energy-dependent term $\sqrt{E^2 - J_\phi^2}$, compared to the giant magnon-like dispersion relation (3.15). It is noted that we need an appropriate factor 1/2 in front of $\sqrt{2\lambda}\Delta\phi_2$ and $\sqrt{2\lambda}\Delta\psi$ to derive the dispersion relations (2.27) and (3.17).

4. Conclusion

Using the specific spherical coordinate representation of CP^3 such that each foliating surface at constant ξ is expressed by the $U(1)$ bundle over $S^2 \times S^2$ with coordinates ψ and (θ_i, ϕ_i) $i = 1, 2$ we have constructed two kinds of string states describing the single giant magnon and single spike solutions with two angular momenta moving in two different subspaces of $R_t \times CP^3$.

In one subspace of CP^3 characterized by one choice $\theta_1 = \theta_2 = \pi/2, \psi = 0$ which satisfy the equations of motion for θ_i and ψ , we have derived the dispersion relation of the single giant magnon specified by the two spins J_1 and J_2 associated with the angular ϕ_1 and ϕ_2 directions. We have observed that the resulting dispersion relation turns out to be the same expression as that of the single big giant magnon solution with two spins [17] derived by the algebraic curve prescription, which reduces to that of the giant magnon in the RP^2 subspace of CP^3 [6] when the finite spin is turned off.

In the other subspace of CP^3 characterized by $\theta_1 = \theta_2 = \pi/2, \phi_1 = \phi_2 \equiv \phi$ which give the complementary solution, we have found some linear relations between the conserved charges and the angle difference with suitable coefficients from which we have determined the dispersion relations of the single giant magnon and single spike solutions specified by the two spins J_ψ and J_ϕ associated with the angular ψ and ϕ directions. When the spin J_ϕ is turned off the dispersion relation of the giant magnon becomes the same expression as that of the single one-spin giant magnon living in the S^2 subspace of CP^3 [5].

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